BBA-003-1164001 Seat No. ______

M. Sc. (Sem. IV) Examination

July - 2021

Mathematics : CMT - 4001

(Linear Algebra)

Faculty Code : 003

Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions:

1) Attempt any five questions from the followings.

2) There are total ten questions.

3) Each question carries equal marks.

1 Answer the following seven: **7 X 2 = 14**

1. Define with example: Algebra over a field.
2. Define with example: Homomorphism between two algebras.
3. Define with example: Invertible Linear Transformation.
4. Define with example: Minimal Polynomial.
5. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x_1, x_2) = (x_1, 3x_2)$.

   Justify, whether 3 is a characteristic root of T or not?
6. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (x_3, x_2, 0)$.

   Justify, whether $W = \{(0,0,z): z \in \mathbb{R}\}$ is invariant under T or not?
7. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x_1, x_2) = (0, x_1)$.

   Justify, whether T would be nilpotent or not?

2 **Answer the following seven:** **7 X 2 = 14**

1. Define with example: Cycle with respect with to a linear transformation.
2. Define Jordan form of a linear transformation.
3. Define with example: Companion matrix.
4. Define with example: Characteristic Polynomial of a linear transformation.
5. State Primary Decomposition Theorem.
6. Let A' denotes the transpose a matrix $A \in \mathbb{F}_n$. Justify, whether $(A')' = A$ or not?
7. State Cramer's Rule.

3 **Answer the following both:** **2 X 7 = 14**

a) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A(V)$. Prove that, T is invertible if and only if the constant term of the minimal polynomial is non-zero.

b) Let V be a finite dimensional vector space over $\mathbb{F}$ and $S, T \in A(V)$ with S invertible. Prove that, $r(ST) = r(TS)$.

4 **Answer the following both:** **2 X 7 = 14**

a) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A(V)$. Let $\lambda \in \mathbb{F}$ be a characteristic root of T. Prove that, λ is a root of a minimal polynomial of T over $\mathbb{F}$.

b) Let V be an n-dimensional vector space over $\mathbb{F}$. Prove that, $T \in A(V)$ is invertible if and only if $m(T)$ is has inverse in $\mathbb{F}_n$.

5 Answer the following both: **2 X 7 = 14**

a) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A(V)$.

Let W be a T-invariant subspace of V. Prove that, T induces a linear transformation $\bar{T}$ on ${}^{V}/_{W}$ defined by $\bar{T}(v + W) = T(v) + W$.

Also prove that, the minimal polynomial of $\bar{T}$ divide the minimal polynomial of T.

b) Let $\mathbb{F}$ be a subfield of a field K. Let $n \in \mathbb{N}$ and $A \in \mathbb{F}_n$. Prove that, A is invertible in $\mathbb{F}_n$ if and only if A is invertible in K_n.

6 Answer the following both: **2 X 7 = 14**

a) Let V be an n-dimensional vector space over $\mathbb{F}$ and $T \in A(V)$. Suppose all the characteristic roots of T lies in $\mathbb{F}$. Prove that, T satisfies a polynomial of degree n over $\mathbb{F}$.

b) Let V be a finite dimensional vector space over $\mathbb{F}$. Let $T \in A(V)$ be nilpotent with index of nilpotence k. Let $v \in V$ be such that, $T^{k-1}(v) \neq 0$. Prove that, the vectors $v, T(v), \dots, T^{k-1}(v)$ are linearly independent over $\mathbb{F}$.

7 Answer the following both: **2 X 7 = 14**

a) State and prove, Cayley-Hamilton Theorem.

b) Let the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathbb{F}_3$. Prove that, A is nilpotent and find the invariants of A.